Ramification of Channel Networks Incised by Groundwater Flow

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Seepage Channels

Re-emerging groundwater can lead to the incision of seepage channels. A particularly compelling example of such channel networks on the Florida Panhandle exhibits characteristic features that offer the ability to test theories for network growth. Our ultimate goal is to understand ramification and growth dynamics of the entire network, so we built a computational model based on the following growth hypothesis:

Channels grow in the direction that captures the maximum water flux. When there are two such directions, tips bifurcate.

Dynamics of Groundwater Flow

By applying Dupuit's approximation (flow driven by hydrostatic pressure) for groundwater flow with constant precipitation R, we obtained a 2D Poisson equation

$$-R = \nabla q, \; q = \kappa h \nabla h$$

$$\nabla^2 h^2 = -\frac{2R}{\kappa} \quad (1)$$

where κ = permeability, R = precipitation rate, h = height of groundwater table.
Close to the springs, rainfall contribution to flux is negligible, leading to Laplace's equation as $R \to 0$

$$\nabla^2 \phi = 0$$

where $\phi = h^2$. The solution can be expanded in the complex plane

$$\phi = -i(a_1 z + i a_2 z^2 + a_3 z^3 + \ldots) \quad (2)$$

where $z = x + iy$. Or, $\phi(\sqrt{iz}) = -i[a_1(iz)^{1/2} + a_2 iz + a_3(iz)^{3/2}]$.

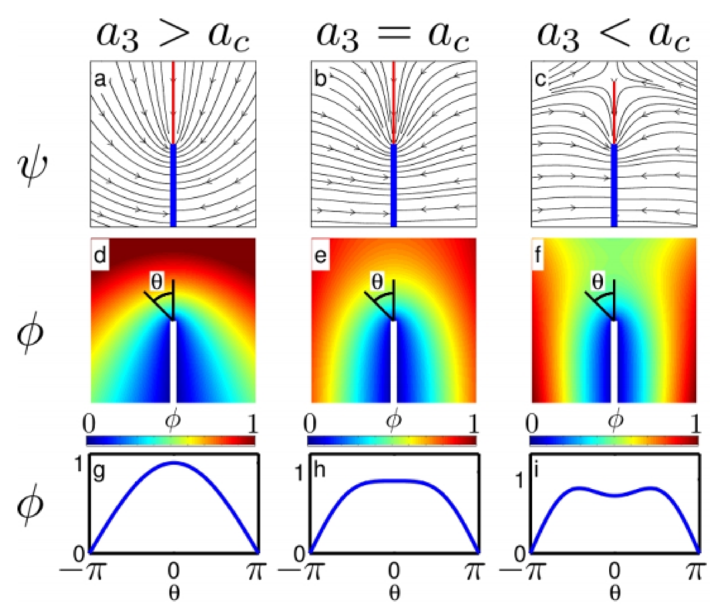
Expansion Coefficients and Growth

The direction of growth can be determined from the expansion of the ground water field around each tip, where each coefficient a_i in this expansion has a physical interpretation.

a_1 determines **groundwater discharge**, leading to a straight growth of the channel

a_2 describes asymmetry in the water field leading to **bending of the stream** in the direction of maximal water flux

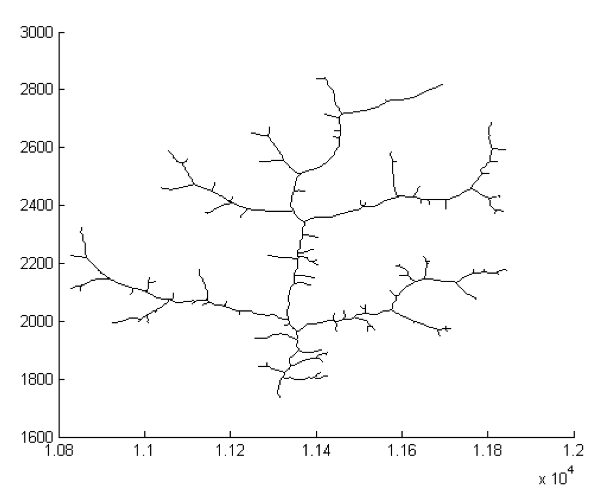
a_1/a_3 determines a **critical distance** r_c over which the tip feels inhomogeneities in the ground water table, which then initiates the splitting of the tip



In order to test our growth hypothesis and to determine r_c, we can grow the Florida network backward.

Backward Growth

Our simulation allows us to determine the significance of our growth hypothesis by comparing a theoretically obtained angle to observation. Here we execute backward channel growth on a small segment of the Florida network.



At each time step, we recalculate the solution of the groundwater field and determine the appropriate expansion coefficients around each tip. We then remove a segment of channel proportional to $\sim a_1$.

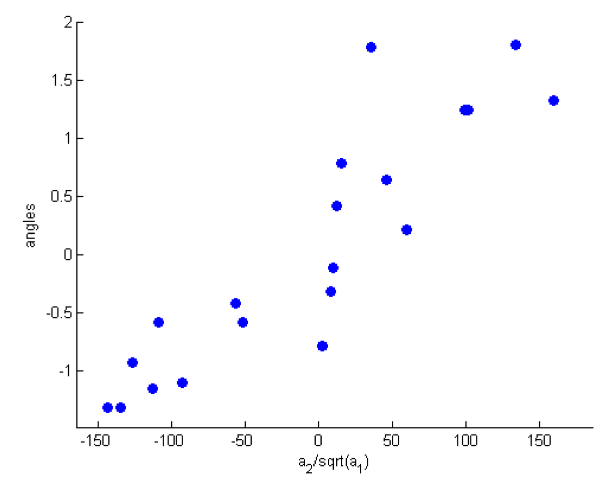
Angles Between Segments

For small angles, we obtain the equation characterizing our streamline by calculating the angle between successive segments

$$\tan\alpha \approx \alpha = \frac{a_2}{a_1}\sqrt{a_1 dt}$$

$$\alpha \sim \frac{a_2}{\sqrt{a_1}} \quad (3)$$

This affords us a metric by which we can compare our data. A scatter plot comparing this approximation for angle to observed tip angles is shown below.



While the trend does appear linear, poor LIDAR resolution created an abundance of points at $\alpha = 0$, which were thrown out, thereby confounding the legitimacy of this trend. Considering this approximation only holds for small angles, it is crucial to extract higher resolution data to further analyze this problem.

Critical Distance r_c

By growing the channels backward, our simulation eventually reached bifurcation points within the network. And by recording the coefficients at the bifurcation, we obtained a characteristic value related to the critical distance r_c.

$$\langle r_c \rangle \sim \langle a_1/a_3 \rangle_{bif} = -3.661$$

compared to a value of $\langle a_1/a_3 \rangle = -1.59$ for the original network. This is consistent with our current understanding of the expansion coefficients. The critical distance increases as the channel tip begins to 'feel' more incongruities in the groundwater table, and this ultimately initiates bifurcation.