Bifurcation of Streams | EP51B

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Seepage Channels

The geometric beauty of channel networks is undeniable!

Pictured above is an example of the geometry we expect when channel growth is largely driven by the relaxation of the underlying groundwater table. Our growth hypothesis:

Channels grow in the direction that captures max water flux.

Darcy's Law in the Dupuit approximation can be written $q \sim -h\nabla h$. If $\nabla q \sim P$ (precipitation), we can rescale and obtain $\nabla^2\varphi = -1$. Near the channels, P $\ll$ groundwater discharge, so:

$$\nabla^2\varphi = 0$$

By associating a point (x, y) with a complex number, $z = x + iy$, $\varphi(x, y)$ can be written as a power series, $\Phi(z) = \sum a_k z^k$. We map this onto an infinite channel using the map $w = \sqrt{iz}$:

$$\Phi(\sqrt{iz}) = -i\left(a_1(iz)^{1/2} + a_2 iz + a_3(iz)^{3/2}\right) + \mathcal{O}(z^2)$$

We surmise that first three coefficients of this expansion manifest themselves in the field as follows:

a_1	**Groundwater Discharge** $\sim$ tip growth velocity
a_2	**Asymmetry Term** $\sim$ stream bending
a_3	**Presence of water table bifurcation**

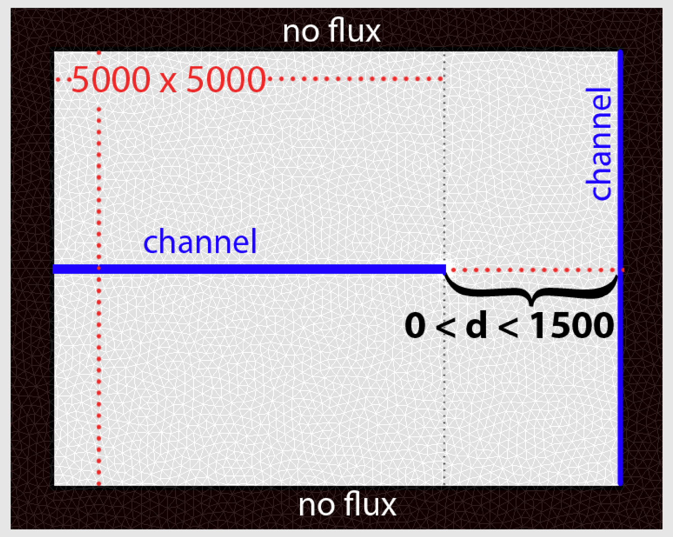
Our hypothesis for a_3 comes observations in the shape of the field. By changing to polar coordinates ($z = re^{i\theta+\pi/2}$), the second derivative of our field function at $\theta = 0$ exhibits an inflection point when

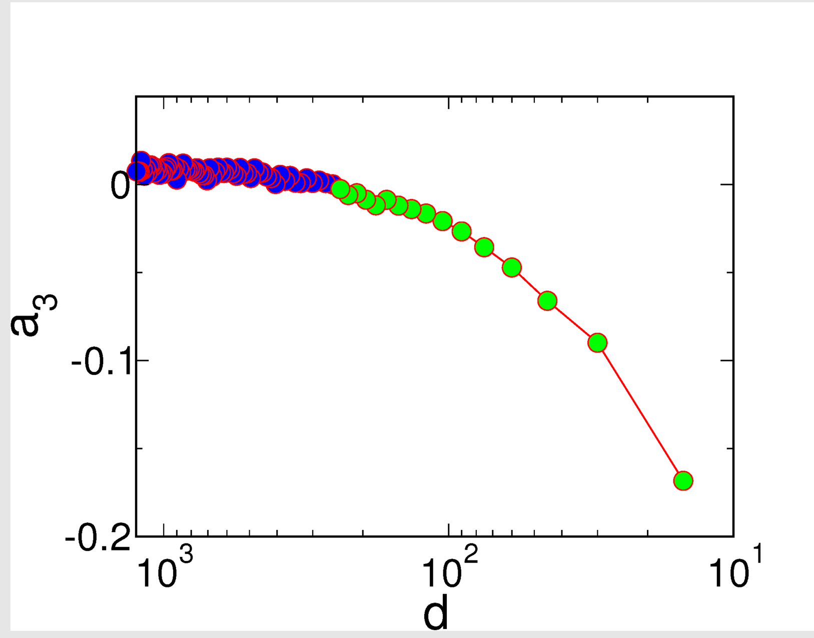
$$-\frac{1}{4}a_1 r^{1/2} - \frac{9}{4}a_3 r^{3/2} = 0$$

a_1 is always positive, and as a result, for $a_3 < 0$, a bifurcation in the water table must exist at some distance r in front of the channel.

Box Growth

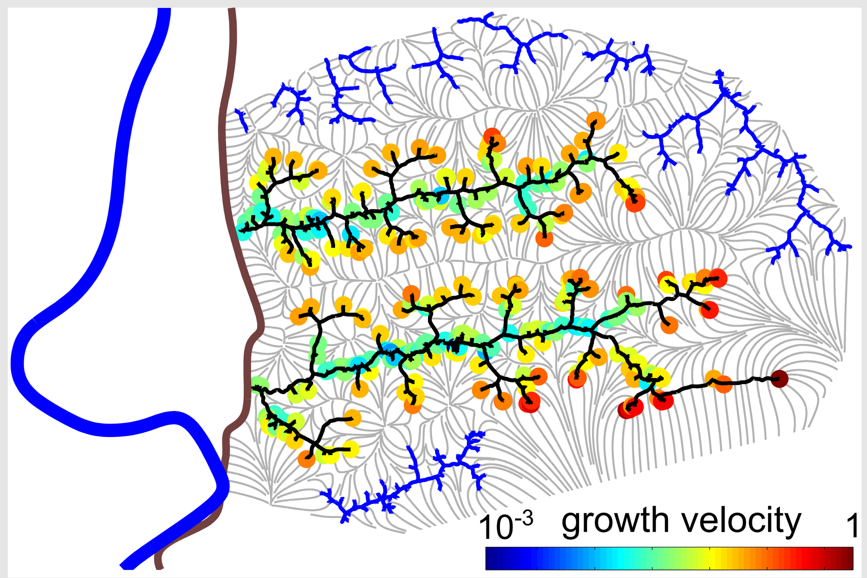
But does a_3 behave as we hypothesize? We create a scenario in which a very long channel grows within a box towards another channel a distance d from its tip. We measured a_3 as a function of d.



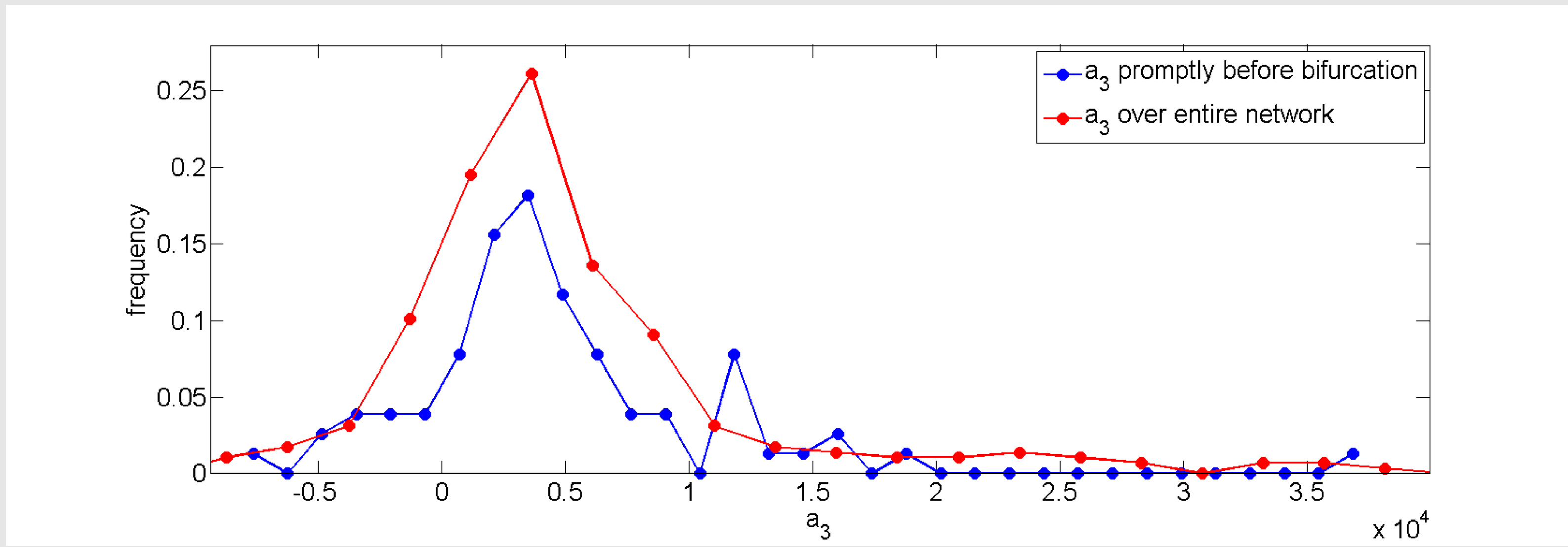


Backward Growth

Does a_3 still behave in Florida? We used LIDAR data to grow the Florida channel heads back along their geometric history at a speed a_1, recalculating a_1 at each time step. Once two branches were fully retracted to their point of bifurcation, a_3 was noted.

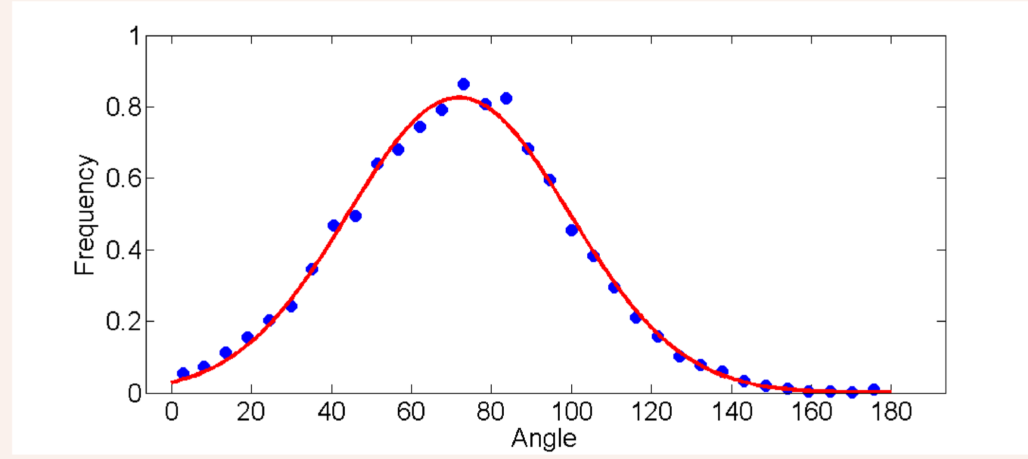


Due to the eclectic nature of our boundary conditions, however, a clear a_3 trend did not manifest in our backward growth record.

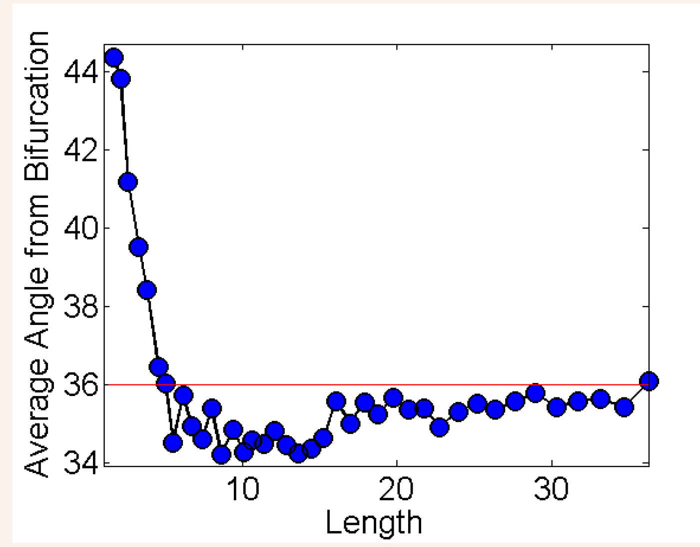


Stream Meandering

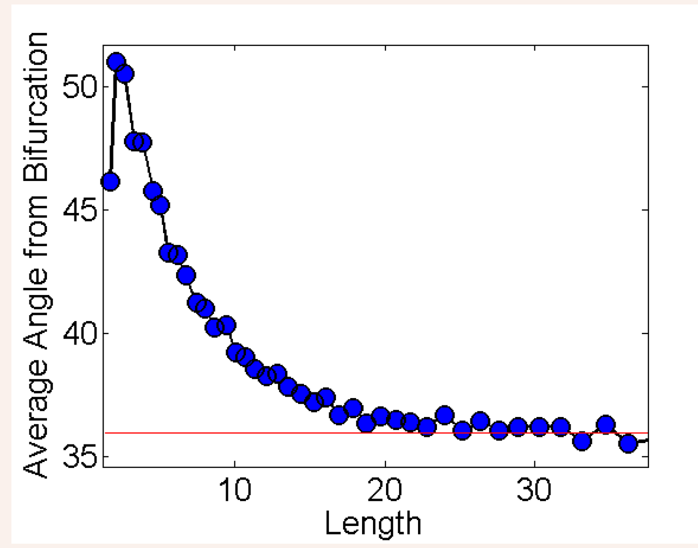
Previous results support the idea that incipient bifurcations split off at a $2\pi/5$ angle.

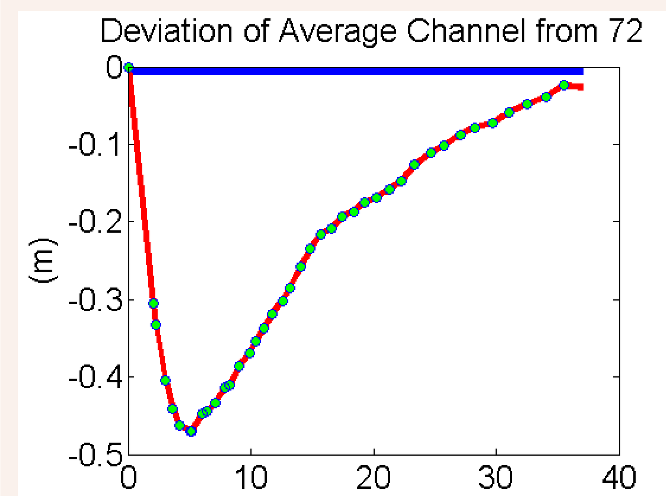


But this relies on our assumption that growth occurs in an infinite plane, and that there exists a systematic length invariance, but this is not always true. Inhomogeneities in the groundwater table have some non-zero effect when $l \sim l_p$, where l is the length of a channel, and l_p is the length of perturbations in the network. But as $l > l_p$, the channel overcomes these inhomogeneities and follows the direction of the streamline entering the tip. We demonstrate this by measuring the average angle versus distance from the point of bifurcation.



The angle starts at roughly $\pi/2$, but shifts below the expected $2\pi/5$ value, implying an initial $\pi/2$ split, and a narrowing as the channel is driven towards $2\pi/5$. To confirm this behavior, we track the average angle the **entire channel** makes with the bisector.





In future work, we hope to combine these results with the "Loewner Equation" to theoretically qualify this trajectory.